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Measurements of the short range diffusion of airborne particles at a height of a few hundred feet in the atmosphere

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J. S. Hay and F. Pasquill

(Meteorological Office)

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SUMMARY

The results of measurements of the vertical distribution of airborne particles, released usually at a height of 500 feet, and sampled for periods of about 30 minutes at downwind distances of 100, 300 and 500 metres, are presented and discussed.

At all distances the frequency distributions of the particle elevation with respect to the site of release are closely similar in shape and size to the frequency distribution of wind inclination at the site of release. This is interpreted as indicating that the diffusion of particles released successively from a fixed point in the atmosphere clear of the ground is dominated by a scale of turbulence for which one can recognise a Lagrangian correlation coefficient (R_ξ) which is maintained close to unity for periods of 1 minute or more.

In contrast to this result the wind observations show that the Eulerian auto-correlation coefficient (R_t) falls to about 0.2 in 10 seconds. Since the present experiments do not reach the condition $R_\xi \leqslant 1$ it is not possible, for example, exactly to compare the respective times T which lead to identity in the quantities

$\frac{1}{T^2}\int_0^T\int_0^t R_\xi \, d\xi \, dt$ and $\frac{1}{T^2}\int_0^T\int_0^t R_{t'} \, dt'dt$.. However it is clear that such an identity will occur with T(Lagrangian) of the order of 10 times T(Eulerian), a result which is qualitatively in keeping with wind tunnel observations by Mickelsen (1955) and with Gifford's (1955) examination of Eulerian and Lagrangian energy spectra at a height of about 300 feet in the atmosphere.

INTRODUCTION

The diffusion of particles released at a fixed point in steady, homogeneous turbulence, conditions which may often occur approximately in the atmosphere clear of the ground, is completely specified by the well known Taylor (1921) expression which may be written in the form:-

$$\overline{z^2} = 2\,\overline{w'^2}\int_0^T\int_0^t R_\xi \, d\xi \, dt \qquad \ldots\ldots\ldots\ldots (1)$$

Here z is the vertical displacement of particles from their mean vertical position after travelling time T, w' is the vertical component of eddy velocity, and R_ξ is the correlation coefficient between the vertical velocity of a particle at one instant and the velocity of the same particle a time ξ later. R_ξ is usually referred to as the Lagrangian correlation coefficient. In practice R_ξ is very difficult, if not impossible, to measure effectively and in applying equation (1) progress has depended (see Sutton 1932, 1934) on specifying intuitively or empirically some relation between R_ξ and measurable quantities. One empirical approach, however, which does not seem to have been exploited to any extent is to use experimental measurements of diffusion to obtain some indications of the

properties of the Lagrangian correlation coefficient in comparison with those of the corresponding, measurable, Eulerian coefficient $R_{t'}$, i.e. the correlation between the velocity of the air at a point at one instant and that at the same point time t' later. The simplest possible type of comparison would be to evaluate

$$\frac{1}{T^2}\int_0^T\int_0^t R_{t'}\, dt'\, dt$$ and to compare with the corresponding double integral

from (1), as a function of T. After commencing the present investigation the writers' attention was drawn to some recent work by Mickelsen (1955) in which precisely this line of approach has been followed in studying diffusion in homogeneous turbulence in a wind tunnel.

The present paper describes experiments in which the vertical distribution of airborne particles released in the atmosphere at a position remote from the ground was examined at various distances downwind up to 500 m. The height of release was usually 500 ft. A description of the technique, which uses apparatus carried on the cables of captive balloons, has been given recently (Hay and Pasquill 1955) and will not be repeated here. Simultaneously, measurements were made of the inclination of the wind at the site of release, in order effectively to specify w' and $R_{t'}$, using a balloon-borne gustiness recorder described previously by Jones (1956).

EQUIPMENT AND TECHNIQUE

In the experiments which are the subject of this paper, the releases of Lycopodium powder lasted normally for 30 min. and the sampling was carried out during the whole period of release. The sampling cylinders were positioned 100 m. downwind of the dispenser in the earlier of these experiments, but as experience was gained this range was extended to 300 and 500 m. The number of heights at which samples were taken was at first 11 but was progressively increased to a maximum of 27. At the central position, four cylinders were exposed to provide data on the reliability of the sampling. During the later experiments, an additional cylinder was exposed at 100 ft., at which height it could normally be expected to be outside the cloud of particles emitted by the dispenser and so give a sample of particles naturally present in the atmosphere.

In addition to wind inclination, measurements of mean horizontal wind speed were made with a vane anemometer incorporated in the gustiness instrument. The temperature difference between the level of the dispenser and a point 200 ft. below was measured using thermistors mounted on the balloon cable and connected to recording apparatus on the ground. The temperature difference between 23 ft. and 4 ft. above the ground was indicated by permanent apparatus located nearby.

To allow correction to be applied for the movement of both balloons, theodolite observations of the elevations of the dispenser and the central sampling position were made at regular intervals. The positioning of the "sampling" balloon downwind of the "source" balloon was directed by an observer stationed about mid-way between the balloons and viewing, in a horizontal mirror, pennants flying from the balloon cables near the central sampling position and dispenser. The former balloon was moved as necessary to keep the pennants in line.

In the assessment of the particle samples, twenty sections of each cylinder were examined by microscope. The instrument used had a magnification of 80 and was masked to give a field of view about 0.12 cm. square, slightly different from that used in the preliminary experiments. The microscopic examination was undertaken by two observers to minimise any personal bias in counting, as the spores sometimes formed only a relatively small proportion of the particles adhering to the cylinders. The mean number of particles counted per 0.12 cm. section of sampling cylinder by each observer was first obtained and the over-all mean taken. At the central sampling position, the results from all four

sampling cylinders were used in finding the mean. A discussion of the statistics of sampling is given in Appendix I. It is confirmed therein that the size of sampling cylinder used, and the method of examination adopted, yielded reasonable estimates of particle deposit, that the Lycopodium spores could be distinguished reliably from other particles and that there were no similar spores naturally present in the atmosphere during the experiments.

REDUCTION AND PRESENTATION OF OBSERVATIONS

(a) Particle distributions

The particle deposits observed in each of the 19 experiments were first plotted against height. In 9 cases it was evident that the cloud of particles had not been adequately sampled, due to insufficient spacing of the sampling cylinders or incorrect positioning of the sampling balloon, and these cases were not considered further. The deposits obtained in the remaining 10 experiments are given in Table I and those for experiment 17 are plotted in Figure 1 as an example.

To facilitate comparison with the measured wind inclinations at the site of release, the vertical distribution of particles was expressed in terms of their angular elevations with respect to the dispenser. It may be noted that the sampling cylinders were actually spaced at equal height intervals but these were so small relative to the distance from the dispenser that they approximated closely to equal elevation intervals. The particle deposit at each level was then expressed as a percentage of the total for all levels in the particular experiment and "cumulative percentage" deposits obtained by addition of successive individual values, commencing with that for the lowest level. For subsequent discussion, the data so derived are presented on probability graph paper in Figures 2 and 3, the "cumulative percentage" deposits being plotted against the elevation mid-way between the level to which they refer and the next higher level. This assumes that the deposit on any cylinder is representative of the interval centred on the cylinder and extending mid-way to the adjacent sampling levels.

In addition to atmospheric diffusion, the relative vertical motion of the dispenser and sampling array due to balloon movement contributes to the observed spread of particles. This contribution actually depends on the difference, ΔH, between the height of a datum point on the sampling array at time T and the height of the dispenser at a time T-t, where t is the time of travel of the particle. It seems unlikely that there will be a high degree of correlation between ΔH and the true vertical displacement of the particle with respect to its point of origin, and an approximate test of this can be made by examining the relation between ΔH and ϕ, where ϕ is the wind inclination at the dispenser position at time T - t. This test was carried out for experiment No.8, which represents the worst case in the sense that the variations of ΔH relative to the apparent vertical displacements of particles were largest, and the above supposition was confirmed. The usual statistical rule for deriving the variance of the difference of two independent quantities may therefore be applied. If σ_s^2 and σ_a^2 are respectively the variances of ΔH and the apparent vertical displacements of the particles, then σ_p^2, the variance of the true vertical displacements, is given by $\sigma_p^2 = \sigma_a^2 - \sigma_s^2$. In practice, these quantities will be expressed as elevations with respect to the dispenser. Values of ΔH were obtained from the theodolite observations, the time t being estimated from the wind speed measured over periods of a few minutes.

(b) Distributions of wind inclination

From the gustiness records, values of the wind inclination ϕ were read off to the nearest 0.5° at $2\frac{1}{2}$ sec. intervals. This proved to be a simple matter when the charts were driven at 3 in./min., but at 1 in./min., the usual speed, it was frequently only possible to estimate upper and lower limits for the value of ϕ because of the cramped nature of the record.

In such cases, the mean of these limits was taken as the appropriate value. Once again, to reduce personal bias as far as possible, the data were extracted independently by two individuals and some compromise reached with readings which differed. Corrections were then applied in the following manner for deflections of the wind-inclination vane which arose from the movement of the supporting balloon rather than from the turbulent motion of the air.

If the elevation α of the vane as observed by the theodolite is small, and changes by an amount $\Delta\alpha$ in a time Δt (usually 10 sec.), the spurious component of wind inclination is $\phi_1 = -\frac{l}{u}\frac{\Delta\alpha}{\Delta t}$ where l is the horizontal distance from the theodolite to the point directly under the vane and u is the horizontal wind speed over the interval Δt. The elevation in fact never exceeded 5°, so that the approximation made in deriving this expression was valid. The distance l was usually within the range 8900 ± 100 ft. but was assumed to have a constant value of 8900 ft. in evaluating ϕ_1, and u was actually meaned over a period of 2 or 3 min. and assumed to apply to the component intervals Δt. It is considered unlikely that this procedure gave rise to an error of more than a few per cent in calculating mean values of ϕ_1. Instantaneous values of ϕ_1 at times corresponding to the readings of ϕ were then estimated by graphical interpolation and used to correct the latter. It will be appreciated that the values of ϕ_1 derived in this way are subject to some uncertainty, especially when they are changing rapidly. They can be accepted with some confidence, however, in eight of the experiments considered here, where ϕ_1 was only 2° or more for 12 per cent or less of the time. In the exceptional cases, ϕ_1 was of this magnitude for 20 and 30 per cent of the time in experiments 8 and 19 respectively.

The corrected values of wind inclination, i.e. $\phi - \phi_1$, from each experiment were formed into a frequency distribution based on class intervals one degree wide and centred on integral values of inclination, the frequencies of the intermediate half-degree values being divided equally between adjacent intervals. These are given in Table II. For comparison with the particle distributions, cumulative frequency distributions were formed for the range of elevations covered by the sampling cylinders. This procedure necessarily excluded the occasional extreme values of inclination which were mainly positive in sign, i.e. indicative of upward moving currents. These distributions are shown together with the corresponding particle distributions in Figures 2 and 3, the cumulative frequencies being plotted against the upper limits of the respective class intervals.

(c) Meteorological and other conditions

The dates and times of the various experiments, and the distances and heights involved, are given in Table III together with wind speed and local and surface temperature gradient measurements. Mean wind speeds at the height of release ranged from 6.1 to 9.5 m./sec. and temperature gradient in the lower layers ranged from slight lapse to slight inversion.

DISCUSSION OF RESULTS

(a) The observed relation between the distributions of particle elevation and wind inclination

The frequency distributions of particle elevation and wind inclination collected in Figures 2 and 3 show three features of interest. Firstly there is the relation between the median values of the two distributions. For seven of the experiments there is agreement to within 1°, in one experiment the particle elevation is higher by 3°, while in the remaining two it is lower by 2°. The mean of the differences in median values, in the sense wind inclination minus particle elevation, is + 0.2° with a standard deviation of 1.5°, in other words there is no significant indication of a declination of the trajectories of the particles with respect to the initial air flow. However, it is noteworthy that this value of

/+ 0.2°

+ 0.2° is very close to that which would follow from the expected terminal velocity of 2 cm./sec. in a wind of about 7 m./sec. In individual experiments the gravitational fall is evidently negligible in comparison with the unknown effects of incomplete sampling (in a statistical sense), and of systematic features such as topographical control on the general flow pattern or variations of eddy structure with height. Nevertheless, the present results indicate that the resultant effect is frequently small and that to a close approximation the median values of particle elevation are equal to the median values of wind inclination at the site of release.

A second feature is that the graphs in Figures 2 and 3 are roughly linear, indicating an approximation to Gaussian form in the distributions of both wind inclination and particle elevation. This is not of particularly novel interest in the case of the wind data, except in so far as they refer to wind inclination at an unusually elevated site, for it is widely thought that turbulent components will usually be so distributed, apart from anistropic motions associated with well developed convective action. In this latter respect it is to be borne in mind that the restriction of our comparison of wind and particle distributions to the height range covered by the sampling apparatus has necessarily excluded the occasional extreme inclinations, most of which are a reflection of this anistropic feature. The main interest is rather in the approximation to Gaussian form in the particle distributions, for here it is to be remembered that the measurements are not made at a single level of say 500 ft., but over a layer which may extend between 200 and 800 ft. and which will contain some systematic variations of wind and eddy structure with height. Inspection of the individual distributions does not suggest any consistent departure from Gaussian form and it is interesting to note that there is no outstanding difference in this respect between results obtained at different distances from the source. In order to give an overall representation of the results the distributions have been normalised to give zero inclination or elevation at 50 per cent and the mean inclinations and elevations at other cumulative frequencies derived. The results are shown in Figure 3 from which it is seen that, on average, the inclination distribution is actually skew with large positive (upwards) values, though the same feature is not evident in the particle distribution.

The third and most interesting feature is the similarity in the dimensions of the wind and particle distributions, though before any realistic comparison can be made it is necessary to correct for the relative vertical displacements of the dispenser and the sampling array, as discussed previously. Values of σ_a, the standard deviation of the apparent elevations of the particles, were extracted from the distributions in Figures 2 and 3 by reading off the elevations at 2 and 98 per cent, i.e. those enclosing 4.12 times the standard deviation in a Gaussian distribution. A similar process was used to obtain σ_w, the standard deviation of the wind inclinations. The standard deviations, σ_s, of the elevations due to the relative vertical displacements, were computed directly. The values of σ_a and σ_s, the resulting values of σ_p, and the corresponding values of σ_w, are collected in Table IV below. Examination of the statistics of sampling (see Appendix I) show that the standard error of σ is of the order of 3 per cent.

/TABLE IV

TABLE IV

Comparison of standard deviations of particle elevation and wind inclination

Expt. No.	Dist. of travel (metres)	Particle elevation σ_s	Particle elevation σ_α (degrees)	Particle elevation σ_p	Wind inclination σ_ω (degrees)	σ_p/σ_ω	$\bar{u}$ (m/sec.)	ΔT °F (23ft.-4ft.)
4	100	1.2	2.1	1.7	3.1	0.55	6.7	-0.6
7	100	1.3	3.9	3.7	3.0	1.23	8.0	+0.2
8	100	2.2	4.1	3.5	2.8	1.25	7.4	0
12	300	0.7	5.0	4.9	4.6	1.07	7.7	-0.4
13	300	0.5	3.9	3.9	3.8	1.03	7.3	+0.6
14	300	1.0	4.2	4.1	4.0	1.03	6.3	-0.4
15	300	0.2	4.9	4.9	4.9	1.00	6.1	-0.6
16	500	0.5	3.0	3.0	3.2	0.94	7.3	-0.7
17	500	0.3	2.9	2.9	2.7	1.07	6.8	-0.3
19	300	0.9	6.1	6.0	4.0	1.50	9.5	+0.2

With the exception of Nos. 4 and 19 there is a remarkable degree of similarity between the values of σ_p and σ_w, irrespective of the distance of travel and the conditions of wind and atmospheric stability prevailing. Taking all experiments σ_p/σ_w has a mean value of 1.07 and a standard deviation of 0.23. The odd result in the case of No.19 is clearly associated with certain subsidiary peaks in the particle distribution, which occur at a height well below the main peak and for which there appear to be insufficient corresponding large negative values of wind inclination. No. 4 was carried out in pre-frontal conditions with drizzle setting in, and is a special case in that an abrupt reduction in gustiness occurred about half way through the experiment. The observation of a distribution of particles which is narrow relative to the distribution of wind inclination could be due to unrepresentative sampling, since the positioning of the sampling array with respect to the cloud of particles was probably controlled more successfully in the second half of the experiment than in the first. This idea is supported by the very high particle deposits which were observed over a sampling time of only 16 min.

The original data have also been independently examined by a statistician. The approximation to Gaussian distribution was confirmed in one case which appeared doubtful by the application of a formal test. In the subsequent treatment, therefore, the data were assumed to be of Gaussian form. Variances of apparent particle elevation (σ_α^2) and of wind inclination (σ_ω^2) were calculated directly from the observations in Tables I and II. The values of σ_α^2 were then corrected as before to give σ_p^2.* Owing to the large number of degrees of freedom involved, the equality of σ_p^2 and σ_ω^2 could not conveniently be tested from the available statistical tables of variance ratio. Instead, a form of "t" test was applied to the differences of the variances. Of the ten experiments, six (Nos. 12, 13, 14, 15, 16, 17) showed no significant difference, while for the remaining four (Nos. 4, 7, 8, 19) the differences were found to be highly significant. However, only in No. 4 is the difference substantial and in the direction of σ_p less than σ_w, and this is the experiment which has already been discussed as possibly anomalous.

/(b)

*The values of σ_p and σ_ω obtained in this fashion differed only slightly from those presented in Table IV.

(b) Interpretation of the results in relation to the Lagrangian and Eulerian correlations in the eddy structure

As stated in the introduction to this paper, the fundamental object of the present investigation was to obtain indirectly some insight into the Lagrangian features of the eddy structure, that is, variations which are related to time in the sense of following the motion, in comparison with Eulerian features, which are related to time at fixed points in space. From Taylor's expression for the scattering of particles (equation 1), it immediately and necessarily follows that a linear variation of standard deviation with time (i.e. in our notation $\sigma_p = \sigma_w$) is held as long as the Lagrangian correlation coefficient remains close to unity. In the simplest possible terms the particles travel for such a time in straight lines, with the velocity imparted to them at the moment of release.

The present results, in showing equality between σ_w and σ_p for distances up to 500 m., imply that the diffusive spread of the particles may be described by equation 1 with R_ξ maintained equal to unity for at least 1 min. This period of maintenance of Lagrangian correlation is considerably longer than that which is required to explain the vertical diffusion of a smoke cloud at ground level. In the latter case (see Sutton, 1949, p.349) the required value of R_ξ falls to about 0.5 in 1 sec. Furthermore, it is very soon obvious from an examination of the wind inclination variation that the apparent persistence of Lagrangian correlation is not associated with a similar feature in the Eulerian correlation. On the contrary, the Eulerian or auto-correlation coefficient computed from corrected values of the wind inclination* at $2\frac{1}{2}$ sec. intervals falls off rapidly, as can be seen from the two cases quoted below in Table V. These figures are the means of three values obtained from analysis of three separate 8 min. periods in each of the two experiments.

TABLE V

Values of auto-correlation coefficient for wind inclination

Time lag (seconds)	$2\frac{1}{2}$	5	$7\frac{1}{2}$	10	15	20	30
Correlation coefficient (Expt. No. 13	0.69	0.40	0.19	0.09	-0.01	0.01	0.02
Correlation coefficient (Expt. No. 17	0.61	0.40	0.29	0.23	0.20	0.14	0.04

/The

* For the purpose of the arguments which are ultimately developed in the present paper it is probably unnecessary to give detailed consideration to the differences between the auto-correlation for inclination ($R_t, (\phi)$) and that for the vertical component of eddy velocity ($R_t, (w)$). However, an analysis of this feature has been carried out (see Appendix II) to confirm that small values of $R_t, (\phi)$ of the order of 0.1 are necessarily associated with similarly small values of $R_t, (w)$.

The simplest possible interpretation of the present results is that, despite the rapid Eulerian variations, the particles actually travel in straight lines for substantial periods. It is clear, however, from practical experience, that this interpretation cannot be correct. Indeed, it is obvious from visual observations of a puff of smoke moving with the wind that the trajectories of individual smoke particles are far from linear, and are in fact rapidly distorted mainly by turbulent motions on a scale smaller than that of the puff itself. Furthermore, we have the evidence of Edinger's (1951) photographic measurements of the trajectories of individual soap bubbles in an isothermal layer at a height of 1000 ft. These provided data for the direct evaluation of the Lagrangian correlation coefficient and showed this falling to 0.5 in about 7 sec.

The real explanation of the present results is thought to be along the following lines. If we consider the particles released over some short time interval, we have in effect the case of a small puff of smoke which moves off downwind, increasing in size as it does so. A continuous release of particles may then be considered equivalent to a series of puffs, each puff leaving the release point with a different vertical velocity (or inclination in the present context) and subsequently growing with downwind travel. At any given distance downwind the vertical distribution of particles, averaged over a period of time, will be a resultant of the growth of the individual puffs and the scattering of their centres of mass. The vertical distributions of particles observed in the present experiments are then entirely explained if, firstly, the puffs travel in straight lines and, secondly, the dimensions of the puff are not more than about half the spread of the centres of mass. For example, if we assume Gaussian distributions, with σ_ω^2 and σ_d^2 respectively the variances of the centres of mass of the puffs (i.e. effectively of the wind inclination) and of the particles within the puff, then for σ_d/σ_ω equal to $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ the resultant σ_p (given by $\sigma_p^2 = \sigma_\omega^2 + \sigma_d^2$) is equal to $1.12\,\sigma_\omega$, $1.05\,\sigma_\omega$ and $1.03\,\sigma_\omega$ respectively.

Independent qualitative evidence for a scattering process of this nature is frequently apparent in the form of the familiar plume of smoke from an industrial chimney. In this case, the plume is usually of somewhat irregular sinuous shape with amplitudes frequently greater than the actual visible width of smoke, but if attention is fixed on some outstanding "element" near the lower or higher points of the instantaneous plume, it will be seen that the element does not trace out the sinuous form, but continues in an apparently straight path inclined downward or upward, for as long as it can be recognised as a coherent puff of smoke. This behaviour has also been observed quantitatively, by one of the writers (F.P.) and J. Crabtree, in the trajectories of smoke puffs emitted successively at a few hundred feet above ground and followed by theodolite over a distance of about 125 m. In the terms of the present measurements the standard deviations of the elevations of the puff centres were constant with distance within a matter of 5 per cent, while the overall vertical spread of the puff centres was certainly greater than the vertical dimensions of the individual puffs by a factor of two or more.

While, on the one hand, it is clearly not enough to consider the Lagrangian correlation coefficient for a <u>particle</u>, and on the other hand it is not physically acceptable to refer to a <u>mass</u> of air, for our elementary puff is in fact not a given mass of air, the features discussed above imply a kind of Lagrangian correlation which is maintained for much longer than the familiar Eulerian correlation at a point. It is as if there is a relatively large scale pattern in the eddy structure which develops and changes quite slowly, but which gives the impression of fairly rapid change merely by virtue of being swept rapidly past an observing instrument. Within this persistent pattern, going up the frequency scale of the spectrum of eddy energ , we have less correlated

/motions

motions which are responsible for the growth of the elementary puff. The interesting point which has yet to be clarified is the length of time for which such a situation can be recognised and how this time is related to the physical conditions.

The greater persistence of Lagrangian as compared with Eulerian correlation has previously been observed in wind tunnel turbulence, and indeed, in an investigation similar in some respects to the present one, Mickelsen (1955) has recently compared the diffusive spread of a gas emitted from a point in wind tunnel flow with that calculated from equation (1), with the Lagrangian correlation coefficient replaced by measured Eulerian values. From this comparison Mickelsen draws certain conclusions regarding the relation between the shapes of the decay curves of the Lagrangian and Eulerian correlation coefficients, though here the argument is rather involved and our impression is that an error has occurred in the transformation from the correlation coefficient to its double integral with respect to time.

In the present discussion we will therefore restrict our attention to a comparison of the forms

$$\frac{1}{T^2}\int_0^T \int_0^t R_\xi \, d\xi \, dt \quad \text{(Lagrangian) and} \quad \frac{1}{T^2}\int_0^T \int_0^t R_{t'} dt' dt \quad \text{(Eulerian)*},$$

when the first of these is given, through equation (1), by the observed value of $\sigma^2/2\overline{v^2}T^2$, and the second may be evaluated from the measurement of the cross-stream component of eddy velocity, v. Equality in the two forms is found to occur when T (Lagrangian) is approximately 3 times T (Eulerian), for a range of values of $\sigma^2/\overline{v^2}T^2$ from 1 to 0.4. The data do not permit evaluation at lower values of $\sigma^2/\overline{v^2}T^2$ but the trend appears to be in the direction of a rapid increase in the ratio of the "Lagrangian" and "Eulerian" times.

In contrast to Mickelsen's experiments, ours do not extend into the region of non-linear relation between σ^2 and t^2, and a similar evaluation of the ratio of the "Lagrangian" and "Eulerian" times for equality in the above forms is accordingly not possible. However, substitution in equation (1) of the auto-correlation data given in Table V leads to a σ, t relation which begins to depart significantly from linearity (i.e. $\sigma^2/\overline{\omega^2}t^2 \simeq 0.8$) for t between 5 and 10 sec., whereas in the observed values a similar order of departure from linearity is not evident even at 70 sec. In other words, this suggests that for equal values in $\sigma^2/\overline{\omega^2}\,t^2$ the "Lagrangian" and "Eulerian" times are in the ratio of about 10 to 1, which is at any rate qualitatively similar to the result obtained in a wind tunnel flow by Mickelsen.

In the observational study of atmospheric turbulence there have recently been some quite novel measurements made by Gifford (1955), in an attempt to compare the Lagrangian and Eulerian characteristics directly at a height of 300 ft. Gifford uses the trajectories of "no-lift" balloons from which to extract vertical velocities of a Lagrangian nature, and compares these on an energy spectrum basis with Eulerian values from a fixed installation. The interesting feature of these results is the suggestion that the Lagrangian spectrum is generally displaced to the <u>low</u> frequency side of the Eulerian spectrum, with the respective frequencies of peak energy in the ratio of 1 to 3 approximately.

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* The notation is that used in the Introduction and differs from that used by Mickelsen.

It seems clear, therefore, that our conclusion regarding the difference in Lagrangian and Eulerian persistence of eddy velocity is supported independently, not only in the sign, but also in the order of magnitude of the difference which we are able to estimate. A more definite evaluation of this difference would certainly be useful in understanding the process of diffusion and it is proposed to continue the work in this direction.

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APPENDIX I TO M.R.P. 1006

DISCUSSION OF SAMPLING TECHNIQUE

In developing the present sampling technique, the first factors to be decided on were the diameter and length of the sampling cylinders. An external diameter of about $\frac{1}{4}$ in. was preferred since glass tubes of this size are fairly robust and their collection efficiency is reasonably high. According to Gregory (Ann. App. Bio., 38, No. 2, 357-376, 1951), it exceeds 40 per cent and does not vary markedly with wind speeds above 3.3 m/sec. As regards length of cylinder, an upper limit is set by the height variation of particle concentration, if the sample is to be representative of a particular level. Assuming the vertical distribution of particle concentration to be Gaussian in form, with standard deviation σ_a, it can be shown that systematic variations in particle deposit along a cylinder of length 1 will at no point exceed 1 per cent if $1 \leq \sigma_a/100$. This requirement was certainly fulfilled here with 1 approximately 4 cm., since the minimum value of σ_a observed was about 5 m. (The cylinders were actually about 5 cm. in length but a section of only 4 cm. was covered with adhesive. The remaining 1 cm. length was covered with sellotape for ease of handling, with an identifying number beneath). To confirm that cylinders of the above dimensions did yield reasonable estimates of particle concentration, four cylinders were exposed 7.5 cm. apart at the central sampling position.

It was of course impracticable to count the total number of particles deposited on a cylinder in one traverse by the microscope, as only a limited (constant) length of cylinder could be viewed. The number of sections scanned on each cylinder was, however, optional up to a maximum of about 34. Assuming the number of particles per section on any cylinder to follow a Poisson distribution (see Gregory, p.363), the standard error of the mean number m from n sections is given simply by $\sqrt{\frac{m}{n}}$ or $\frac{100}{\sqrt{mn}}$ when expressed as a percentage of the mean. For a constant degree of accuracy of assessment therefore, the number of sections examined should be increased as the deposit decreases. For simplicity, a fixed number of sections, $n = 20$, was actually used, so that the mean deposit m per section may be expected to have a standard error of 20 per cent of m or less for $m \geq 1$. In each experiment, smaller deposits were in fact observed on several cylinders at the top and bottom of the sampling array. It was also necessary to consider how far the estimate of the deposit on a cylinder was affected by personal bias, as it was not possible to identify every spore with absolute certainty. The precaution was therefore taken of having each set of sampling cylinders scrutinized by two individuals.

Selecting at random a number of sampling cylinders, the standard errors of the mean deposits m were calculated in the usual manner and compared with the values of $\sqrt{\frac{m}{20}}$ using a form of "chi-square" test. For cylinders with m ranging up to 30, these tests showed that the values were in good agreement, confirming that, in these cases, the particles were distributed along each cylinder according to a Poisson distribution. In two cases, however, in which m was much larger, namely 70.60 and 94.35, there was a significant difference, $\sqrt{\frac{m}{20}}$ giving an underestimate of the true standard error by a factor of about 2. To reduce the computational work in subsequent comparisons, the standard error of m was assumed to be $\sqrt{\frac{m}{20}}$ in all cases.

Considering the cylinders exposed 7.5 cm. apart at the same level, there were only three cases in which th deposit on an individual cylinder differed significantly (at the 5 per cent level) from the overall mean for the level in the same experiment. In these instances the discrepancy was within 15 per cent so it may be concluded that cylinders of the size used and examined in the manner described yield reasonable estimates of the particle deposit. Comparing now the results obtained by different observers, there

/were

were only about 2 per cent of cases in which the two estimates of deposit differed at the 5 per cent level of significance. This would appear to indicate that the spores could usually be distinguished reliably from other particles adhering to the cylinders.

As indicated above, the mean deposit m on some cylinders was less than unity so that the standard error of m would exceed 20 per cent of m in these instances. Such errors, however, are unlikely to have been in the same sense for all cylinders in one experiment so that a smaller error in the standard deviation σ_a of the vertical distribution of particles might be anticipated. This was actually confirmed for experiments 8, 13 and 17. Assuming the standard error of each m to be $\frac{m}{\sqrt{20}}$, the standard error of σ_a was computed and, in the worst case, was found to be only about 3 per cent of σ_a. It is unlikely that this figure would have been greatly exceeded, if at all, in any of the other experiments.

Finally, reference may be made to the cylinders which were exposed at 100 ft. in seven of the experiments in order to give samples of particles naturally present in the atmosphere. In four experiments, no spores were observed and there were no deflections of the wind vane corresponding to the elevation of the cylinder. In the other three experiments, however, a few spores were observed but the wind inclination records in these cases showed occasional values corresponding to the elevation of the cylinder. There seems little doubt, therefore, that there were no spores naturally present in the atmosphere during these experiments, and that the observed distributions were in fact of particles emitted by the dispenser.

APPENDIX II TO M.R.P. 1006

THE EULERIAN AUTO-CORRELATION COEFFICIENT OF THE VERTICAL COMPONENT OF EDDY VELOCITY

In the present experiments the vertical gustiness recorder gives wind inclination $\phi = \tan^{-1}\frac{w}{u} \simeq \frac{w}{u}$,* where w is the vertical component of velocity and u the horizontal component in the direction of the mean wind. The purpose of the present analysis is to investigate the error involved in calculating the auto-correlation coefficient directly from the fluctuations $\phi' (= \phi - \overline{\phi})$ instead of from the values of w', for the derivation of which corresponding values of u' (as distinct from $\overline{u}$) would be required,

since
$$\phi \simeq \frac{w}{u} = \frac{\overline{w} + w'}{\overline{u} + u'} . \qquad (1)$$

Using suffixes r and s = r + t to denote values at two instants of time t apart,

$$\phi_r = \frac{w_r}{u_r} , \qquad \phi_s = \frac{w_s}{u_s}$$

and
$$w_r w_s = u_r \phi_r u_s \phi_s$$

Taking mean values and expanding, it follows that

$$\overline{\omega_r' \omega_s'} = \overline{u}^2 \, \overline{\phi_r \phi_s} \, (1 + \alpha) - \overline{w}^2$$

where
$$\alpha = \frac{\overline{u_r' \phi_r \phi_s}}{\overline{u}\,\overline{\phi_r \phi_s}} + \frac{\overline{u_s' \phi_r \phi_s}}{\overline{u}\,\overline{\phi_r \phi_s}} + \frac{\overline{u_r' u_s' \phi_r \phi_s}}{\overline{u}^2 \overline{\phi_r \phi_s}} \qquad (2)$$

and similarly, in steady conditions,

$$\overline{\omega_r'^2} = \overline{\omega_s'^2} = \overline{u}^2 \, \overline{\phi^2} \, (1 + \beta) - \overline{w}^2$$

where
$$\beta = \frac{\overline{2 u' \phi^2}}{\overline{u}\,\overline{\phi^2}} + \frac{\overline{u'^2 \phi^2}}{\overline{u}^2 \overline{\phi^2}} \qquad (3)$$

The Eulerian auto-correlation coefficient, R(w), for the vertical component is therefore,

$$\frac{\overline{\omega_r' \omega_s'}}{\overline{\omega'^2}} = \frac{\overline{\phi_r \phi_s}}{\overline{\phi^2}} \cdot \frac{\left(1 + \alpha - \frac{\overline{w}^2}{\overline{u}^2 \overline{\phi_r \phi_s}}\right)}{\left(1 + \beta - \frac{\overline{w}^2}{\overline{u}^2 \overline{\phi^2}}\right)} \qquad (4)$$

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* The ϕ used here is true wind inclination and hence corresponds to $\phi - \phi_1$ in the text.

The corresponding correlation coefficient, $R(\phi)$, for wind inclination, is

$$\frac{\overline{\phi'_r \phi'_s}}{\overline{\phi'^2}} = \frac{\left(\frac{\overline{\phi_r \phi_s}}{\overline{\phi^2}} - \frac{\bar{\phi}^2}{\overline{\phi^2}} \right)}{\left(1 - \frac{\bar{\phi}^2}{\overline{\phi^2}} \right)}$$

and for the values of $\frac{\bar{\phi}^2}{\overline{\phi^2}}$ which obtained for the data discussed in Table V of the text (i.e. approximately 0.01 and 0.001 in Nos. 13 and 17 respectively)

$$\frac{\overline{\phi_r \phi_s}}{\overline{\phi^2}} = R(\phi) .$$

The terms α and β in equation (4) involve correlations between the u and w components which are unobtainable from the present data and we are only able to assign limits to α and β, as follows. For the term $\overline{u'_r \phi_r \phi_s}$ we may write

$$\left|\overline{u'_r \phi_r \phi_s}\right| \leq \overline{\left|u'_r \phi_r \phi_s\right|} \leq \overline{|u'_r| |\phi_r \phi_s|} .$$

If now we assume that the maximum numerical value of u'_r is three times its standard deviation

$$\left|\overline{u'_r \phi_r \phi_s}\right| \leq 3\sqrt{\overline{u'^2}}\ \overline{|\phi_r \phi_s|} .$$

For small values of r-s = t, ϕ_r and ϕ_s will usually be of the same sign and as a further approximation the modulus on the R.H.S. may be omitted and we may write

$$\left| \frac{\overline{u'_r \phi_r \phi_s}}{\bar{u}\ \overline{\phi_r \phi_s}} \right| \leq \frac{3\sqrt{\overline{u'^2}}}{\bar{u}} ,$$

and similarly for

$$\frac{\overline{u'_s \phi_r \phi_s}}{\bar{u}\ \overline{\phi_r \phi_s}} \quad \text{and} \quad \frac{\overline{u' \phi^2}}{\bar{u}\ \overline{\phi^2}} . \qquad (5)$$

We are unable to specify the variation of a term of the form $\overline{u'_r \phi_r \phi_s}$ with increasing time-lag t, but since we have no reason to expect it to <u>increase</u>, we will assume the inequality in (5) to apply to all values of t. By

the same general argument we write

$$\left|\overline{u'_r u'_s \phi_r \phi_s}\right| \leqslant 9\,\overline{u'^2}\;\overline{\phi_r \phi_s}\ ,$$

hence

$$\left|\frac{\overline{u'_r u'_s \phi_r \phi_s}}{\overline{u}^2\;\overline{\phi_r \phi_s}}\right| \leqslant \frac{9\,\overline{u'^2}}{\overline{u}^2}$$

and similarly for $\dfrac{\overline{u'^2 \phi^2}}{\overline{u}^2\,\overline{\phi}^2}$ (6)

For the present experiments we may take

$$\frac{\overline{u'^2}}{\overline{u}^2} \simeq \frac{\overline{w'^2}}{\overline{u}^2} \simeq .0005$$

so that from (2), (3), (5), and (6)

$$|\alpha| \leqslant 0.47$$

and

$$|\beta| \leqslant 0.47$$

In (4) the term $\dfrac{\overline{w}^2}{\overline{u}^2\,\overline{\phi_r \phi_s}} \simeq \dfrac{\overline{\phi}^2}{R(\phi)\,\overline{\phi}^2}$

$$\simeq \frac{0.01}{R(\phi)} \quad \text{for No. 13}$$

and $$\simeq \frac{0.001}{R(\phi)} \quad \text{for No. 17}$$

Similarly $\dfrac{\overline{w}^2}{\overline{u}^2\,\overline{\phi}^2} \simeq 0.01$ for No. 13

$$\simeq 0.001 \quad \text{for No. 17}$$

and may be neglected.

When $R(\phi) \simeq 1$ (i.e. $r-s$ = t small) the terms α and β must be of the same sign and equal in magnitude and then from (4) we have the obvious result that $R(w) = R(\phi)$. Otherwise we can only specify outside limits to $R(w)$ and, for example, when $R(\phi) = 0.1$ it follows from (4) and the quantities evaluated above that

$$0.03 \leqslant R(w) \leqslant 0.26.$$

Although this process does not allow us to specify precise magnitudes for $R(w)$ it does mean that small values of $R(\phi)$ of the order of 0.1 can be taken to imply similarly small values of $R(w)$.

TABLE I

Particle deposits on sampling cylinders

Height above central sampling cylinder (metres)	No. of particles per 0.12 cm. section of sampling cylinder			Height above central sampling cylinder (metres)	No. of particles per 0.12 cm. section of sampling cylinder						
	Experiment No.				Experiment No.						
	4	7	8		12	13	14	15	16	17	19
+ 26	-	1.90	2.35	+ 78	0.60	0.55	0.15	0.45	-	0.05	-
+ 24	-	2.15*	4.10*	+ 72	0.65	0.45	0.25	1.00	0.25	0.15	0.45
+ 22	-	2.45	5.85	+ 66	0.80	0.75	0.25	0.80	1.10	0.15	1.40
+ 20	0	4.95*	10.00*	+ 60	0.65	0.35	0.30	2.05	0.55	0.15	1.10
+ 18	-	7.45	14.15	+ 54	0.90	0.65	0.75	4.55	1.00	0.05	1.25
+ 16	0.30	10.80*	16.35*	+ 48	1.30	0.95	0.65	5.40	0.55	0.35	1.85
+ 14	-	14.15	18.50	+ 42	2.30	2.15	1.05	5.65	0.80	0.55	1.35
+ 12	0.90	23.55	31.55	+ 36	2.00	1.45	0.85	6.50	0.85	0.85	1.55
+ 10	-	16.65	30.55	+ 30	1.60	1.45	1.15	5.15	0.75	1.15	3.60
+ 8	9.65	20.35	24.50	+ 24	3.50	4.60	1.85	6.20	1.25	0.85	6.10
+ 6	-	10.90	24.20	+ 18	4.50	12.95	5.25	6.80	1.05	2.05	9.35
+ 4	93.70	6.70	18.30	+ 12	8.05	13.25	7.65	6.20	2.90	3.60	9.95
+ 2	-	7.05	14.60	+ 6	6.10	23.45	7.80	4.10	3.35	2.70	11.60
0	74.15	3.55	7.10	0	7.80	22.15	11.10	8.30	5.30	4.45	11.90
- 2	-	5.20	4.00	- 6	6.90	14.05	3.85	7.00	4.95	6.65	14.75
- 4	10.80	4.30	3.90	-12	5.50	8.05	2.95	4.85	6.65	7.45	12.70
- 6	-	0.35	3.00	-18	3.65	2.85	8.45	3.85	5.45	5.85	8.75
- 8	4.60	0.80	1.70	-24	7.30	5.85	6.05	2.90	3.60	4.65	6.70
-10	-	0.80	1.25	-30	3.30	4.90	4.85	2.25	3.35	4.45	5.50
-12	0.70	0.35	0.20	-36	2.45	0.95	0.45	2.55	3.70	2.05	1.35
-14	-	0.05	0.20	-42	0.75	0.10	1.25	1.60	0.50	2.30	0.80
-16	0.30	0*	0.20*	-48	0.35	0.45	0.35	0.20	0.15	2.65	1.65
-18	-	0	0.15	-54	0.30	0.10	0.05	0.30	0.20	1.65	2.90
-20	0	0*	0.10*	-60	0.20	0.05	0	0.10	0.20	1.25	1.20
-22	-	0	0.05	-66	0.05	0.05	0	0.35	0.20	0.85	0.45
-24	-	0*	0*	-72	0.10	0.05	0	0.10	0.15	0.35	3.60
-26	-	0	0	-78	0	0	0	0	0.10	0.25	2.45

* Interpolated values.

TABLE II

Frequency distributions of wind inclination

Corrected inclination (degrees)	Frequency of occurrence — Experiment No.									
	4	7	8	12	13	14	15	16	17	19
+ 29	1.0	–	–	–	–	–	–	–	–	–
+ 28	–	–	–	–	–	–	–	–	–	–
+ 27	–	–	–	–	–	–	–	–	–	–
+ 26	–	–	–	–	–	–	1.0	1.0	–	–
+ 25	–	–	–	–	–	–	1.5	1.0	–	–
+ 24	–	–	–	1.0	–	–	1.5	–	–	–
+ 23	–	–	–	–	–	–	–	–	–	–
+ 22	–	–	–	–	0.5	–	–	1.5	–	–
+ 21	–	–	–	–	0.5	–	2.0	0.5	–	–
+ 20	–	–	–	1.0	–	–	1.5	0.5	–	–
+ 19	–	–	–	1.5	–	–	1.5	1.0	–	–
+ 18	–	–	–	3.5	0.5	0.5	4.5	0.5	1.0	–
+ 17	–	–	–	1.5	0.5	0.5	4.0	2.0	1.0	–
+ 16	1.0	–	–	5.5	–	0.5	8.0	4.5	–	–
+ 15	3.0	–	–	3.0	–	0.5	7.5	3.0	1.5	1.0
+ 14	–	–	–	6.5	–	0.5	5.0	4.5	0.5	–
+ 13	0.5	–	–	7.0	1.5	4.0	11.0	7.0	0.5	1.5
+ 12	3.0	–	–	13.0	2.5	5.0	7.0	12.5	1.0	1.5
+ 11	2.5	1.5	1.0	15.5	4.5	3.0	10.0	13.0	1.0	–
+ 10	3.0	2.0	2.0	25.5	5.0	5.5	15.5	15.5	2.5	1.5
+ 9	4.0	5.0	2.5	16.0	15.5	6.0	9.0	15.0	5.0	3.0
+ 8	4.5	4.5	2.5	28.5	22.5	13.0	15.5	11.0	3.0	6.0
+ 7	12.0	4.0	4.5	25.5	23.0	10.0	13.5	11.0	3.5	7.0
+ 6	12.5	7.5	16.5	23.0	21.5	10.0	24.5	14.5	6.0	6.0
+ 5	13.0	11.5	32.5	22.0	24.0	10.5	28.0	20.5	18.5	6.5
+ 4	19.5	27.0	41.5	35.0	47.0	26.5	32.0	15.5	23.0	17.5
+ 3	23.5	41.5	68.5	54.0	65.0	36.0	47.5	22.5	37.5	25.0
+ 2	41.5	49.0	101.5	71.5	97.0	49.0	44.0	28.0	90.0	37.5
+ 1	55.5	99.5	129.0	81.0	129.5	68.5	68.0	56.0	116.0	54.5
0	82.5	121.0	113.5	106.0	154.0	91.0	45.0	70.0	114.0	61.0
– 1	53.0	140.0	79.0	100.0	118.0	102.0	57.0	88.5	106.0	70.0
– 2	12.5	102.5	53.0	87.0	75.5	87.5	69.0	104.5	78.0	96.0
– 3	5.0	44.0	35.5	84.5	56.5	51.5	52.0	72.5	57.5	106.0
– 4	2.0	32.0	17.0	49.5	37.5	50.5	40.0	56.0	24.0	101.5
– 5	3.5	13.5	8.5	44.0	21.5	26.0	25.5	31.5	13.0	75.0
– 6	1.5	7.0	8.0	21.5	9.5	17.0	24.0	15.5	9.5	54.5
– 7	1.0	2.0	1.5	9.0	11.0	13.0	11.5	15.5	4.0	36.0
– 8	–	1.0	1.0	6.5	4.0	4.0	8.5	3.0	2.0	21.5
– 9	–	1.0	1.0	3.5	3.0	2.0	7.0	1.0	0.5	12.0
– 10	–	0.5	–	2.0	2.0	1.0	5.0	–	–	12.0
– 11	–	1.5	–	1.5	–	–	3.0	–	–	4.0
– 12	–	–	–	–	0.5	–	–	–	–	–
– 13	–	–	–	–	1.5	–	–	–	–	–
– 14	–	–	–	–	–	–	3.5	–	–	–
– 15	–	–	–	–	–	–	1.5	–	–	–

TABLE III

Summary of meteorological and other conditions

Experimental Number	4	7	8	12	13	14	15	16	17	19
Date	16 September	26 September	26 September	4 October	4 October	13 October	14 October	17 October	17 October	28 October
Time G.M.T.	0828–0844	1436–1506	1606–1636	1521–1601	1636–1716	1522–1552	1425–1455	1151–1221	1430–1500	1517–1557
Distance of sampling (metres)	100	100	100	300	300	300	300	500	500	300
Height of dispenser above ground (metres)	143	147	255	140	143	145	154	150	151	129
Height of central sampling cylinder relative to dispenser (metres)	-0.2	-6.2	-9.0	-0.6	-4.3	-11.7	-14.4	-13.2	-8.2	-5.4
Wind speed (m./sec.)	6.7	8.0	7.4	7.7	7.3	6.3	6.1	7.3	6.8	9.5
Local temperature gradient*	-	0.90 Γ	-	0.66 Γ	0.62 Γ	0.82 Γ	0.86 Γ	0.87 Γ	0.76 Γ	-
Surface temperature gradient (23ft. - 4ft.) (°F)	-0.6	+ 0.2	0	-0.4	+ 0.6	-0.4	-0.6	-0.7	-0.3	+ 0.2
Cloud amount and height (feet)	8/8 As. 3/8 Sc 2500	3/8 Sc.5000 2/8 Cu.3000	5/8–7/8 Sc.3500	5/8 Sc.5000 2/8 Cu.3000	6/8 Sc.5000 2/8 Cu.3000	2/8 Ci.Cs. 1/8–3/8 Cu.2500	4/8 Sc.2500	6/8 Ac.As. 2/8–3/8 Cu.Sc.3000	3/8 Cu.Sc. 3000	2/8–4/8 Sc.3000

* Obtained from measurements over a height interval of 200 feet near the dispenser. Γ = dry adiabatic lapse rate.

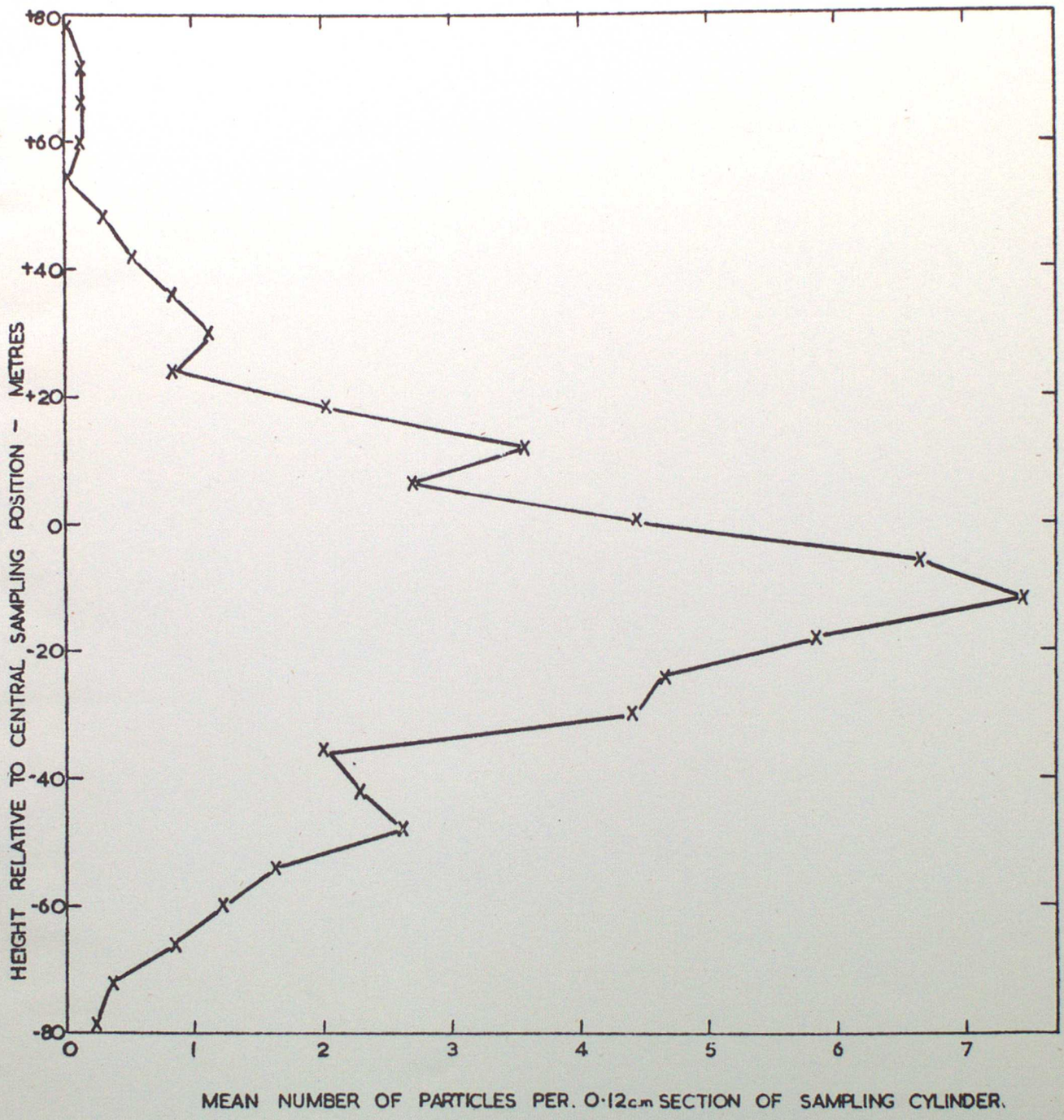

FIG. 1 VERTICAL DISTRIBUTION OF PARTICLES IN EXPERIMENT 17.

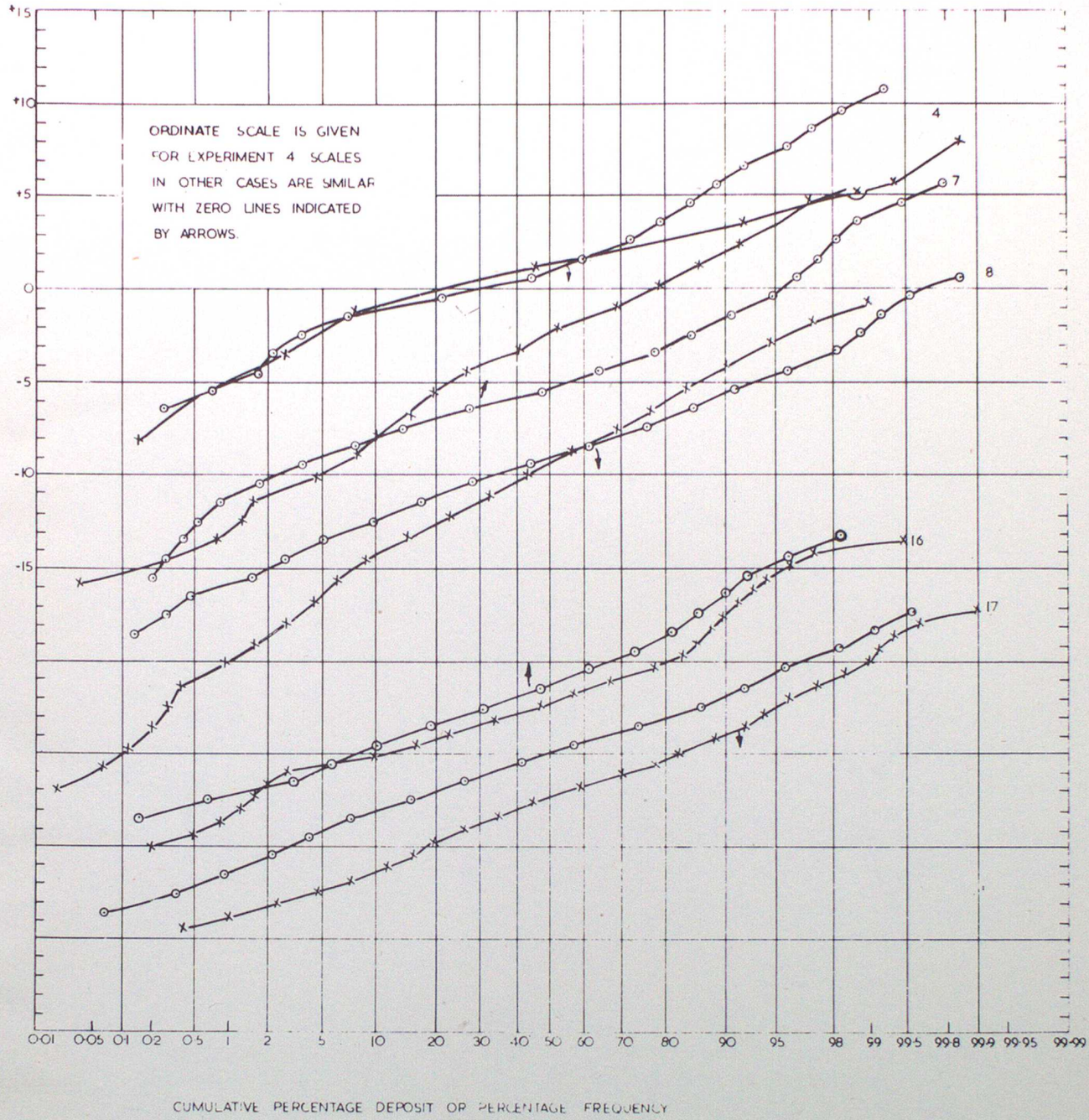

FIG 2. DISTRIBUTIONS OF PARTICLE ELEVATION (✕) AND WIND INCLINATION (⊖) FOR EXPERIMENTS 4, 7, 8, 16 AND 17

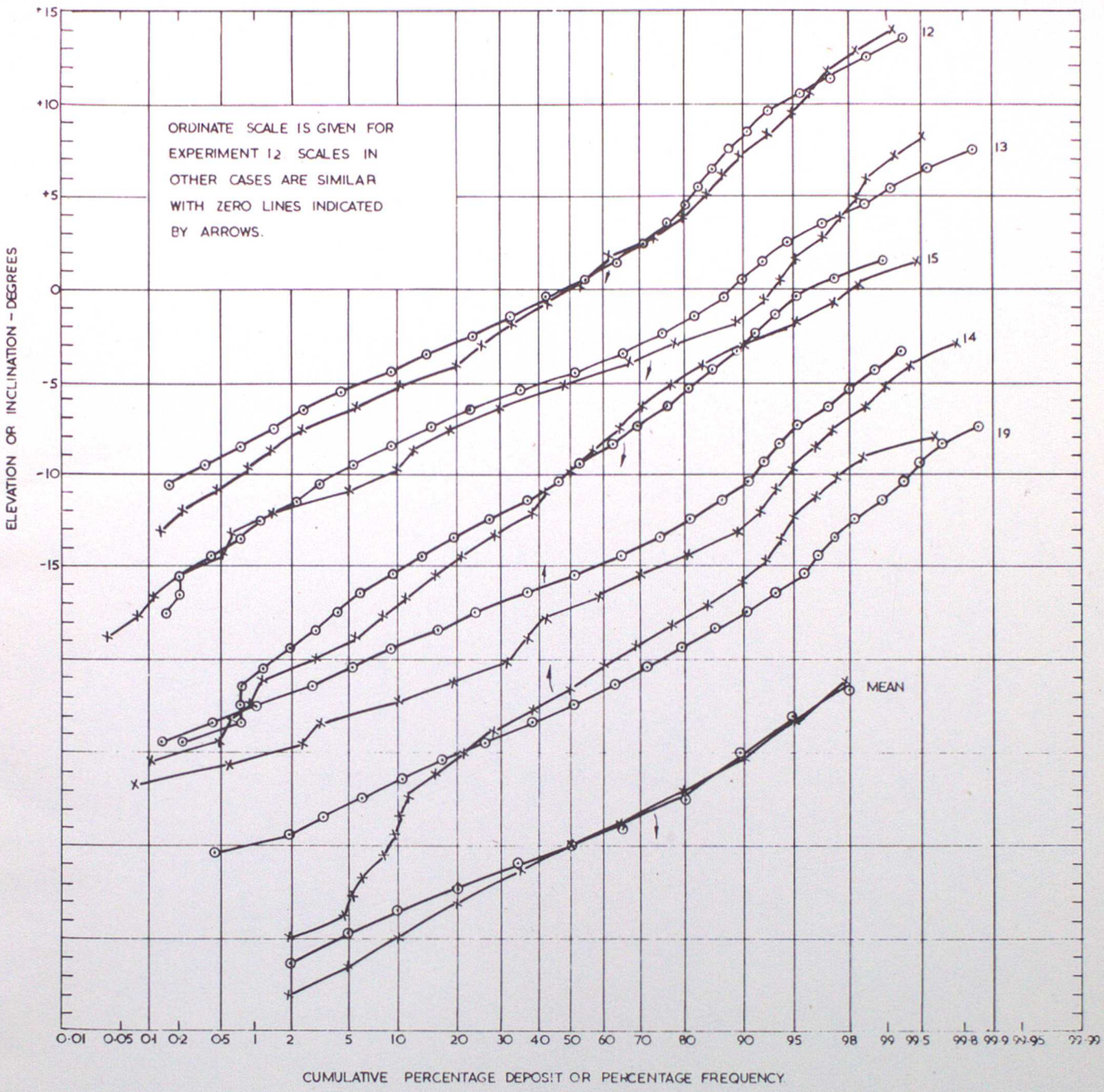

FIG. 3 DISTRIBUTIONS OF PARTICLE ELEVATION (⊛) AND WIND INCLINATION (⊖) FOR EXPERIMENTS 12, 13, 14, 15 AND 19 AND MEAN DISTRIBUTIONS FOR ALL EXPERIMENTS INCLUDING THOSE IN FIG. 2.